21. Evaluate $\int_0^2 \dfrac{dx}{x^2 + 4}$ using Romberg's method and obtain an approximate value of π.

22. Using fourth order Runge–Kutta method, evaluate the value of y when $x = 1.1$, given that $\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{1}{x^2}$, $y(1) = 1$.

23. (a) The first four moments of a distribution about $x = 2$ are 1, 2.5, 5.5 and 16. Calculate the four moments.

   (i) about the mean

   (ii) about zero.

   (b) A random variable X has the pdf $f(x) = \dfrac{c}{1 + x^2}, -\infty < x < \infty$. Determine c and the distribution function.

24. (a) Distinguish multiple correlation and partial correlation.

   (b) Explain contingency tables.

$\underline{4}$ BCA $\rightarrow$ Numerical and statistical Methods

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its absolute and relative errors.

2. Write Newton–Raphson formula and its order of convergence.

3. If $A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$, prove that $A + A' = 0$.

4. What is the geometrical significance of trapezoidal rule?

5. Define initial value problem. Give an example.

6. Write second order Runge–Kutta formula.

7. What is the algebraic sum of the deviation of a set of n values from their arithmetic mean?

8. What is the variance of the first 7 natural numbers?

9. If A and B are independent events, are $\overline{A}$ and $\overline{B}$ independent? Justify your answer.

10. Write the mean and variance of binomial distribution.

11. What are the normal equations to fit the curve $y = bx^a$?

12. Write the formula for F–test.

SECTION B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Find a root of the equation $x^3 - 3x + 1 = 0$ lying between 1 and 2 correct to three decimal places, using bisection method.

14. Solve the following system of equations by Gauss–Jordan method.

$x + y + z = 9$

$2x - 3y + 4z = 13$

$3x + 4y + 5z = 40$

15. Explain cubic spline method to compute first and second derivatives of a function.

16. Using Milne's predictor corrector method, find $y(0.4)$ for the differential equation $\frac{dy}{dx} = 1 + xy$, $y(0) = 2$.

17. Find the arithmetic mean $\bar{x}$, standard deviation σ and percentage of cases within $\bar{x} \pm \sigma$ in the following frequency distribution.

Marks :	10	9	8	7	6	5	4	3	2	1
Frequency :	1	5	11	15	12	7	3	3	0	1

18. X can hit a target 3 times in 5 shots; Y can hit 2 times in 5 shots; Z can hit 3 times in 4 shots. They fire a volley. What is the probability that 2 shots hit the target?

19. Two variables x and y have the regression lines $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Find

(a) the mean values of x and y.

(b) the correlation coefficient between x and y.

(c) the variance of y, if the variance of x in 25.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Solve the following system of equations by LU–decomposition method.

$2x - 6y + 8z = 24$

$5x + 4y - 3z = 2$

$3z + y + 2z = 16$