

23. Evaluate $\iint_D (x^2 + y^2) dx dy$ where D is the region bounded by $y = x^2$, $x = 2$ and $y = 1$.

24. Find the Fourier series expansion for $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$
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50402/SBAMA

Time : Three hours

Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

- Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$ given that $1 + \sqrt{-1}$ is a root of it.
- If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find Σx^2 .
- Change the equation $2x^4 - 3x^3 + 3x^2 - x + 2 = 0$ into another the coefficient of whose highest term will be unity.
- Define characteristic root and characteristic vector.
- Verify Cayley-Hamilton theorem $A = \begin{bmatrix} 1 & 7 \\ -1 & 2 \end{bmatrix}$.
- Find characteristic values of $A = \begin{bmatrix} 4 & -4 \\ 2 & 7 \end{bmatrix}$.
- Write down the expansion of $\tan 4\theta$.

$$A = \begin{bmatrix} 0 & 0 & 5 \\ 2 & 1 & 4 \\ 1 & 0 & -1 \end{bmatrix}$$

14. Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2.

15. Verify Cayley-Hamilton theorem for

$$17 \quad \begin{bmatrix} 45 & -16 \\ -6 & \end{bmatrix}$$

Answer any FIVE questions.

PART B — (5 × 5 = 25 marks)

$$0 < x < \pi$$

12. Find b_n in the Fourier series of $f(x) = -x$ for

$$11. \text{ Evaluate } \int_{\frac{3}{2}}^{2} \left(\frac{3}{2} \right).$$

$$10. \text{ Evaluate } I = \int_{4\sqrt{3}}^{8} xy \, dx \, dy.$$

9. Show that $1 - \tanh^2 x = \sec^2 x$.

$$8. \text{ Evaluate } \lim_{\theta \rightarrow 0} \frac{\cot \theta + \cot^2 \theta}{\cot 3\theta}.$$

16. Expand $\sin^6 \theta$ in series of cosines of multiples of θ .

$$17. \text{ Prove that } \tanh 3x = \frac{\tanh^3 x + 3\tanh x}{1 + 3\tanh^2 x}.$$

18. Find the Jacobian for

$$x = u \sin a + v \cos a$$

$$y = u \cos a - v \sin a$$

19. Determine the Fourier expansion of

$$f(x) =$$

$$\pi^2 - x^2, -\pi < x < \pi.$$

PART C — (3 × 10 = 30 marks)

20. Calculate a root between 1 and 2 of $x^3 - 3x + 1 = 0$ using Horner's method.

21. Find the characteristic vectors of

$$17 \quad \begin{bmatrix} 45 & -16 \\ -6 & \end{bmatrix}$$

22. If $\tan(A+iB) = x+iy$, prove that

$$(a) x^2 + y^2 + 2x \cot 2A = 1$$

$$(b) x^2 + y^2 + 1 - 2y \cot h 2B = 0.$$

13. The roots of the equation $8x^3 - 14x^2 + 7x - 1 = 0$ are in geometric progression. Find them.

14. Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2.

15. Verify Cayley-Hamilton theorem for